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THE FLEXURAL RESPONSE OF A SUBMERGED OR FLOATING BODY TO AN UNDERWATER EXPLOSION

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PART I - THEORY

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by George Chertock, Ph.D

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ABSTRACT

An underwater explosion forms a gas bubble which pulsates, generates a changing pressure field, and induces flexural vibrations in a floating or submerged ship. Equations are derived which couple the motion in each normal flexural mode to the pulsation of the bubble. Some special cases of target response are discussed. The analysis is shown to be applicable if the duration of the compressive phase, the shock wave, is small compared with the pulsation period of the bubble and with the flexural period of the target. Equations are also derived which give the mode shapes and frequencies for free vibration in water in terms of these modes and frequencies for free vibration in air.

INTRODUCTION

When an explosive charge is detonated under water, the detonation products form a gas bubble which pulsates and migrates in the water, and thus induces a changing pressure field. These pressures, in turn, can act upon a submerged or floating target, causing bodily motions and elastic vibrations. There already exists an extensive literature which describes both experimental and theoretical investigations into the motions of the gas bubble, but without any reference to its effect upon an elastic target.[1]

This report presents a theoretical analysis of how a slender target, such as a ship or submarine, reacts to the pressures. The purpose of the investigation was to establish the physical mechanism responsible for the flexural motions and to show quantitatively how these motions depend upon the elastic properties of the target and upon the size, position, and motion of the gas bubble. The method used in the analysis is to separate the target reactions and water motions into normal modes of motion and then to show how each mode is coupled to the bubble pulsation. The effects of the initial shock wave are at first omitted, but these effects are included in a later stage of the analysis.

Although the theory may seem to be unduly mathematical and divorced from practical problems, the applications of this theory are significant and will be discussed in subsequent reports.

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[1] References are listed on page 20.

NORMAL MODES OF THE BODY

We shall consider only small vertical motions of an elongated body for which the instantaneous displacement is uniform for all points in the same transverse section. Such motions are generated by rigid vertical displacements, by rigid rotations about a horizontal axis perpendicular to the length, and also by combined bending and shear vibrations in planes perpendicular to the length. For a body which is long in comparison with its width or depth, there exists a set of natural modes of motion of this type, i.e., motions which it may have when moving freely, in vacuum, without applied external forces. These motions may be described by specifying the vertical displacement $\psi(x,t)$ of a longitudinal axis of the body as a function of the time t and the distance x from one end.

$$\psi(x,t) \propto \psi_i(x)\cos\alpha_i t \qquad i = 0,1,2\cdots \qquad [1]$$

The functions $\psi_i(x)$ describe the mode shapes. The index i, which labels the various mode functions, may conveniently be taken as equal to the number of nodes of $\psi_i(x)$ over the length of the body. The frequencies α_i are the natural circular frequencies in air. These mode shapes and frequencies completely specify the flexural properties of the body and will be taken as known functions. In practice, they may be determined by calculating the eigenfunctions and eigenvalues of some differential equation which governs the flexure, or perhaps by experiment.

In general the mode functions ψ_i are orthogonal over the length of the target, with $m(x)$ the mass per unit length as a weighting function, so that

$$\int_0^l \psi_i\,\psi_j\,m\,dx = M_i\,\delta_{ij} \qquad i,j = 0,1,2\cdots \qquad [2]$$

where M_i is defined as the generalized mass associated with the i^{th} mode. Also these mode functions form a complete set, so that the most general dynamic configuration of the axis, even in forced motion, can be expressed as a superposition

$$\psi(x,t) = \sum_i q_i(t)\,\psi_i(x) \qquad [3]$$

✓ where $q_i(t)$ are a set of generalized coordinates. For these motions the instantaneous kinetic energy of the body can be given as

$$T_s = \sum_i \frac{M_i}{2}\dot{q}_i^2 \qquad [4]$$

and the instantaneous potential energy as

$$V_s = \sum_i \frac{M_i}{2}\alpha_i^2 q_i^2 \qquad [5]$$

In order to illustrate the mode functions, we may consider the special case of a uniform bar of length l. Then $\psi_0 = 1$ is the mode function for a rigid vertical translation, $\psi_1(x) = 1 - \frac{2x}{l}$ denotes the mode shape for rotation, $\psi_2(x)$ denotes the mode shape when the bar is flexing in two-noded vibration, the center moving out of phase with the two ends, etc. Also M_0 denotes the mass of the bar, M_1 denotes the moment of inertia of the bar about its midlength, etc.

MOTION OF THE WATER

Consider now the motion of the water. We assume that the flow is incompressive and inviscid, and that a velocity potential $\phi(\xi,\eta,\zeta,t)$ exists which satisfies the following conditions:

$$\nabla^2\phi = 0 \quad \text{at interior points } \xi,\eta,\zeta \text{ of the fluid} \tag{6}$$

$$\nabla\phi = 0 \quad \text{at infinity} \tag{7}$$

$$-\frac{\partial\phi}{\partial n} = v_g \quad \text{on the surface of the bubble} \tag{8}$$

where n is an outward normal and v_g is the local normal velocity of the bubble outward; and

$$-\frac{\partial\phi}{\partial n} = \sum_i \dot{q}_i\,\psi_i \cos(n,k) \quad \text{on the surface of the body} \tag{9}$$

where (n,k) is the angle between the vertical and the outward normal to the surface. Also the pressure must be continuous across the bubble surface so that, if $p_g(t)$ is the internal pressure in the bubble (assumed uniform), then at its surface

$$p_g = p_h + \rho\frac{\partial\phi}{\partial t} \tag{10}$$

where p_h is the hydrostatic pressure at the point and ρ is the water density. Equations [6] through [10] plus an equation of state for the gas bubble completely specify a boundary value problem for ϕ. This function may now be constructed by superposition of several different potentials as follows.

First there is the velocity potential ϕ_g which is due to the motion of the gas bubble when the elastic body is replaced by an immovable rigid surface and the bubble is constrained to maintain its original motion. Then ϕ_g is a harmonic function whose gradient vanishes at infinity, and

$$-\frac{\partial \phi_g}{\partial n} = v_g \qquad \text{on the surface of the bubble } G \qquad [11]$$

$$-\frac{\partial \phi_g}{\partial n} = 0 \qquad \text{on the surface of the target } S \qquad [12]$$

Second there are the velocity potentials

$$\Phi_i = \dot{q}_i(t)\,\phi_i(\xi,\eta,\zeta) \qquad i = 0,1,2 \qquad [13]$$

Each potential describes the flow when the body is vibrating in a prescribed mode and the bubble is absent. Thus ϕ_i is a harmonic function whose gradient vanishes at infinity and

$$-\frac{\partial \phi_i}{\partial n} = \psi_i \cos(n,k) \text{ on } S \qquad [14]$$

With these definitions it is easily verified that

$$\Phi = \Phi_g + \sum_i \dot{q}_i\,\phi_i \qquad [15]$$

satisfies Equations [6] through [10] except perhaps for [8], the boundary condition on G. In order to satisfy this last condition it would be necessary that

$$\frac{\partial \phi_i}{\partial n} = 0 \text{ on } G \qquad [16]$$

a condition which has not been prescribed in order to avoid making ϕ_i depend on t. However, in the further analysis we shall suppose that the bubble is sufficiently far from the body that the values of ϕ_i at G are small and $\frac{\partial \phi_i}{\partial n}$ at G is negligible.

Now we can write the kinetic energy of the water as an integral over the entire volume of water in terms of the potentials Φ_g and Φ_i and then transform the result, by means of Green's theorem, into integrals over the surface of the solid S and the surface of the bubble G.

$$T_L = \frac{\rho}{2}\int(\nabla\Phi)^2\,d\tau = \frac{-\rho}{2}\int_S \Phi\frac{\partial\Phi}{\partial n}\,d\sigma - \frac{\rho}{2}\int_G \Phi\frac{\partial\Phi}{\partial n}\,d\sigma \qquad [17]$$

Substituting for Φ from [15] and using the boundary conditions [11], [12], and [14]

$$T_L = \frac{\rho}{2}\int_S \Phi_g\Big[\sum_i \dot{q}_i \psi_i \cos(n,k)\Big]d\sigma + \frac{\rho}{2}\int_S \Big(\sum_i \dot{q}_i \phi_i\Big)\Big[\sum_j \dot{q}_j \psi_j \cos(n,k)\Big]d\sigma$$

$$+ \frac{\rho}{2}\int_G \Phi_g v_g \, d\sigma - \frac{\rho}{2}\int_G \Phi_g\Big(\sum_i \dot{q}_i \frac{\partial \phi_i}{\partial n}\Big)d\sigma \qquad [18]$$

$$+ \frac{\rho}{2}\int_G \Big(\sum_i \dot{q}_i \phi_i\Big) v_g \, d\sigma - \frac{\rho}{2}\int_G \Big(\sum_i \dot{q}_i \phi_i\Big)\Big(\sum_j \dot{q}_j \frac{\partial \phi_j}{\partial n}\Big)d\sigma$$

Some of these terms can be consolidated by means of a relation deduced from Green's second theorem.

$$\int_S \Big(\Phi_g \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial \Phi_g}{\partial n}\Big)d\sigma = -\int_G \Big(\Phi_g \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial \Phi_g}{\partial n}\Big)d\sigma \qquad [19]$$

$$\int_S \Phi_g \psi_i \cos(n,k)\,d\sigma = \int_G \Phi_g \frac{\partial \phi_i}{\partial n}\,d\sigma + \int_G \phi_i v_g \, d\sigma \qquad [20]$$

which may be substituted in the first term in [18].

Consider now the last integral in [18]. The factor ϕ_i is approximately uniform over G if the bubble is sufficiently distant from the solid. Also $\frac{\partial \phi_i}{\partial n}$ averages to zero over G. Hence this last integral can be neglected compared, for example, to the preceeding term.

We consider now the integrand of the last term in [20]. The function ϕ_i describes the flow which would prevail if the bubble were removed. Hence its mean value over the surface of G is equal to the value which would exist at the center of G. However, the presence of the bubble modifies this flow by adding a component which is essentially a dipole term proportional to the magnitude of ϕ_i. The mean value of this induced component over G will be zero. Also, if the bubble is sufficiently distant from S, the bubble must be essentially spherical, and v_g will be approximately uniform over the surface. Hence, we can write the last term in [20] as

$$\int_G \phi_i v_g \, d\sigma \simeq \phi_{ig} \frac{dV_g}{dt} \qquad [21]$$

where V_g is the volume of the bubble and ϕ_{ig} is the value for ϕ_i which would exist at the position of the center of G, if the bubble were absent.

Likewise, since v_g is almost uniform over G, the third integral in [18] can be transformed

$$\frac{\rho}{2}\int_G \Phi_g v_g \, d\sigma \simeq \frac{\rho}{2}\Phi_{gg}\frac{dV_g}{dt} \tag{22}$$

where Φ_{gg} is a suitable mean value of Φ_g over G. Finally we can further simplify the terms in [18] by defining an entrained mass of water

$$\begin{aligned} L_{ij} &= -\rho\int_S \phi_i \frac{\partial \phi_j}{\partial n}\, d\sigma = -\rho\int_S \phi_j \frac{\partial \phi_i}{\partial n}\, d\sigma = L_{ji} \\ &= \rho\int_S \phi_i \psi_j \cos(n,k)\, d\sigma = \rho\int_S \phi_j \psi_i \cos(n,k)\, d\sigma \end{aligned} \tag{23}$$

The symmetry property of L_{ij} follows from Green's first equation.

Substituting [20], [21], [22], and [23] into [18] and rearranging terms, the kinetic energy of the water now becomes

$$T_L = \sum_{i,j}\frac{L_{ij}}{2}\dot{q}_i\dot{q}_j + \frac{\rho}{2}\Phi_{gg}\dot{V}_g + \sum_i \rho\Phi_{ig}\dot{V}_g\dot{q}_i \tag{24}$$

There are two terms which contribute to the potential energy of the bubble and water. First there is the internal energy change of the bubble in its adiabatic expansion. This depends upon the volume and upon the equation of state for the gas; it shall be denoted simply by $E(V_g)$. Second there is the potential energy of the water which has been displaced by the expanding bubble. This energy must equal $p_h V_g$. There are also additional potential energy terms which are due to the migration of the bubble in a field in which pressure gradients exist. Such gradients will be due to gravity and to the presence of surfaces. However, these additional terms will be neglected in order to simplify the analysis. This means that the further analysis is applicable only to those times during which bubble migration is negligible.

Adding all terms now, we have for the kinetic and potential energies of target, water, and bubble,

$$T = \sum_i \frac{M_i}{2}\dot{q}_i^2 + \sum_{i,j}\frac{L_{ij}}{2}\dot{q}_i\dot{q}_j + \frac{\rho}{2}\Phi_{gg}\dot{V}_g + \sum_i \rho\Phi_{ig}\dot{V}_g\dot{q}_i \tag{25}$$

$$V = \sum_i \frac{M_i}{2}\alpha_i^2 q_i^2 + p_h V_g + E(V_g) \tag{26}$$

EQUATIONS OF MOTION OF THE SUBMERGED BODY

We can now apply Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0 \qquad [27]$$

and obtain the differential equation for the flexural motion

$$M_i \ddot{q}_i + \sum_j L_{ij} \ddot{q}_j + \rho \phi_{ig} \ddot{V}_g + M_i \alpha_i^2 q_i = 0$$

$$(M_i + L_{ii})(\ddot{q}_i + \omega_i^2 q_i) + \sum_{j \neq i} L_{ij} \ddot{q}_j = -\rho \phi_{ig} \ddot{V}_g \qquad [28]$$

where

$$\omega_i^2 = \frac{M_i}{M_i + L_{ii}} \alpha_i^2 \qquad [29]$$

It appears that each mode of motion behaves like a linear oscillator which is inertia-coupled to each of the other modes. The generalized mass for the i^{th} mode has been augmented by an entrained mass of water L_{ii}, and this increased mass is also effective in decreasing the apparent natural frequency according to Equation [29]. The generalized force, the term on the right in Equation [28], is the product of a time-dependent factor $\ddot{V}_g$, which depends only on the bubble, and a space factor ϕ_{ig}, which depends only on the flexural properties of the body and the position of the bubble. An alternate form of this generalized force can be obtained from Equations [20] and [21], whence the generalized force becomes

$$Q_i = -\int_S \rho \dot{\phi}_g \psi_i \cos(n,k)\, d\sigma + \int_G \rho \dot{\phi}_g \frac{\partial \phi_i}{\partial n}\, d\sigma \qquad [30]$$

Now ϕ_g is approximately uniform over G, while $\frac{\partial \phi_i}{\partial n}$ averages to zero over G. Hence the last term is negligible. Also $\rho\dot{\phi}_g$ is the pressure due to the bubble motion when the target is clamped rigidly. Hence the generalized force can be computed as though the body were rigid.

The coefficients $L_{ij}\,(i \neq j)$ measure how much the inertia of the water couples together two modes which had been independent for vibrations in air. These coefficients may vanish if there is sufficient symmetry. For example, suppose the target is symmetric about a vertical transverse plane through the midlength. Then the functions ψ_i will be symmetrical or antisymmetrical about this midplane depending on whether i is even or odd. Likewise ϕ_i must have the same symmetry as ψ_i. Hence for such a body, whenever i and j are of opposite parity, the coupling coefficient L_{ij} must vanish, or

$$L_{ij} = \rho \int \phi_i \psi_j \cos(n,k)\, d\sigma = 0 \qquad [31]$$

For the more general case of a body of arbitrary shape we can transform to a new set of coordinates and mode shapes so that in these new coordinates there will be no coupling coefficients. Thus, return to Equations [25] and [26] and transform to a new set of coordinates $q'_r(t)$ so that in terms of these new coordinates the kinetic and potential energy terms which are independent of the bubble become simultaneously sums of squares.

$$\sum_i M_i \dot{q}_i^2 + \sum_{i,j} L_{ij} \dot{q}_i \dot{q}_j = \sum_r \dot{q}_r'^2 \qquad [32]$$

and

$$\sum_i M_i \alpha_i^2 q_i^2 = \sum_r \omega_r'^2 q_r'^2 \qquad [33]$$

Since the kinetic energy is a positive definite form, this can be accomplished by a real linear transformation:

$$q_i(t) = \sum_r a_{ir} q'_r(t) \qquad [34]$$

Now if we define

$$\psi'_r = \sum_i a_{ir} \psi_i \qquad [35]$$

$$\phi'_r = \sum_i a_{ir} \phi_i \qquad [36]$$

and substitute in [25] and [26], we get from Lagrange's equation

$$\ddot{q}'_r + \omega_r'^2 q'_r = -\rho \phi'_{rg} \dot{V}_g \qquad [37]$$

where ϕ'_{rg} is the value of ϕ'_r at the position of the center of G. The details of the transformation are shown in the Appendix. The final result is that each mode of motion for the combined system of water and body acts independently as a simple linear oscillator.

The function ϕ_{rg}' cannot be expressed analytically as a function of the bubble position because of the generality of the boundary conditions. However, we can derive a useful approximation which is applicable over a large range of particular cases. This approximation applies to submerged bodies when (1) the distance from the charge to a point on the target surface is large compared with the cross-sectional dimensions of the target, and also,

(2) the body is symmetric about a horizontal plane containing its longitudinal axis (since we are considering vertical motions).

We write ϕ_{ig}' in the familiar form of Equation [38] below, in terms of the values of ϕ_i' and $\partial\phi_i'/\partial n$ at S, r_g denoting the distance from G to a point on S, see Figure 1.

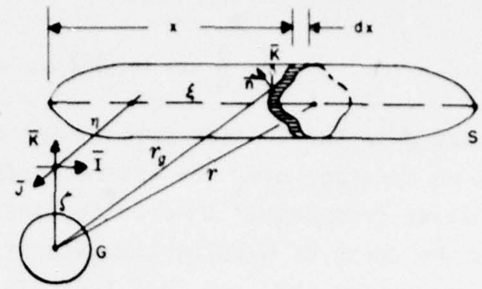

Figure 1 - Geometry for Submerged Body

$$4\pi\phi_{ig}' = \int_S \phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) d\sigma - \int_S \frac{1}{r_g}\frac{\partial\phi_i'}{\partial n}\, d\sigma \qquad [38]$$

Now cut the body into disks of width dx by transverse planes perpendicular to its axis and let ds be an element of the perimeter around the disk at x. Then, since $-\dfrac{\partial\phi_i'}{\partial n} = \psi_i' \cos(n,k)$,

$$4\pi\phi_{ig}' = \int_0^l \oint \phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) ds\,dx + \int_0^l \psi_i' \oint \frac{1}{r_g}\cos(n,k)\, ds\,dx \qquad [39]$$

Consider the special case for which r_g is much larger than the cross section of the disk, and let r be the distance from G to the axis of the disk. Then

$$\oint \phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) ds = -\oint \phi_i' \frac{\vec{n}\cdot\vec{r}_g}{r_g^3}\, ds \simeq -\oint \phi_i' \frac{\vec{n}\cdot\vec{r}}{r^3}\, ds \qquad [40]$$

or, in terms of the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$, in the directions of the ξ, η, ζ axis

$$\oint \phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) ds = -\frac{\xi}{r^3}\oint \phi_i'\left(\vec{n}\cdot\vec{i}\right) ds - \frac{\eta}{r^3}\oint \phi_i'\left(\vec{n}\cdot\vec{j}\right) ds - \frac{\zeta}{r^3}\oint \phi_i'\left(\vec{n}\cdot\vec{k}\right) ds \qquad [41]$$

Because of the assumed symmetry about the horizontal midplane, $(\vec{n}\cdot\vec{i})$ and $(\vec{n}\cdot\vec{j})$ must both be symmetric about this midplane, while ϕ_i' must be antisymmetric about the same section. Hence the first two integrals must vanish. Also the integrand of the third term in [41] can be transformed by Equations [83] and [87] of the Appendix, so that [41] reduces to

$$\oint \phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) ds = \frac{\zeta}{r^3}\frac{m}{\rho}\left(\psi_i' - \psi_i''\right) \qquad [42]$$

where ψ_i'' is a particular combination of the ψ_i' functions as described in the Appendix. Now, returning to the second term in [39],

$$\oint \frac{1}{r_g} \cos(n,k)\, ds\, dx = \int div\, \frac{\vec{k}}{r_g}\, dA\, dx = -\frac{\zeta}{r^3} A\, dx \qquad [43]$$

where A is the section area of the disk and the left side may be considered as an integral over the entire surface of the disk. The second integral follows from Gauss' theorem applied to the volume of the disk. Note that ζ is the depth of G below the axis of S, and if G is above S, then ζ is negative. Substituting [42] and [43] into [39]

$$4\pi\phi'_{ig} = -\frac{\zeta}{\rho}\int_0^l \frac{m}{r^3}\,\psi''_i\, dx - \frac{\zeta}{\rho}\int_0^l \frac{A\rho - m}{r^3}\,\psi'_i\, dx \qquad [44]$$

This last result is a generalization of an analysis given by Lamb for the effect at a distance due to the rigid translation of a solid in a liquid.[2]

Finally, substituting this value for ϕ'_{ig} in [37]

$$\ddot{q}'_i + \omega'^2_i\, q'_i = \left[\frac{\zeta}{4\pi}\int_0^l \frac{m\psi''_i}{r^3}\, dx + \frac{\zeta}{4\pi}\int_0^l \frac{A\rho - m}{r^3}\,\psi'_i\, dx\right]\ddot{V}_g \qquad [45]$$

Note that the transformation between q' and q is the inverse of that between ψ' and ψ as detailed in the Appendix. In many practical cases, the body displaces its own weight of water, so that

$$\int_0^l (A\rho - m)\, dx = 0,$$

and we should expect the second integral on the right to be small compared with the first. In this equation, all the parameters may be determined experimentally by observing the free vibrations of the target in water and the gas bubble in a free field.

PROPORTIONAL BODY

The simplest version of this analysis occurs in the special case where the density of the body is uniform and equal to $\rho = \frac{m}{A}$ and where the body is "proportional." We define a proportional body as one for which

$$\oint \phi_i \cos(n,k)\, ds = a_i\, m\psi_i \qquad [46]$$

where a_i is a constant, independent of x. Multiply the equation by ψ_j and integrate over the length of the body:

$$\int_0^l \psi_j \oint \phi_i \cos(n,k)\, ds\, dx = a_i \int_0^l m\psi_i\, \psi_j\, dx$$

Therefore

$$L_{ij} = a_i \rho M_i \delta_{ij} \qquad [47]$$

Hence a proportional body is one for which the cross coupling coefficients of [28] vanish, and the transformation to new coordinates is trivial, i.e.,

$$q_i' = q_i\,(M_i + L_{ii})^{\frac{1}{2}} \qquad\qquad \psi_i' = (M_i + L_{ii})^{-\frac{1}{2}}\,\psi_i$$

$$\omega_i' = \omega_i \left(\frac{M_i}{M_i + L_{ii}}\right)^{\frac{1}{2}} \qquad\qquad \psi_i'' = \frac{(M_i + L_{ii})^{\frac{1}{2}}}{M_i}\,\psi_i$$

and substituting in [45],

$$\ddot{q}_i + \omega_i^2\, q_i = \left[\frac{\rho\zeta}{4\pi M_i} \int_0^l \frac{A\,\psi_i}{r^3}\,dx\right] \ddot{V}_g \qquad [48]$$

The entrained mass L_{ij} has disappeared from the equation except for its effect on the natural frequency. This notion of a proportional body is not merely a mathematical abstraction but it is a useful concept in practice where the mode function in water is experimentally the same as the mode function in air.

A simple consequence of this applies to the vertical translation of the solid. This may be considered a mode for which $\psi_i = \psi_0$, a constant, and $\omega_0 \to 0$. The terms in [48] now no longer have any dependence upon the elastic properties of the body, and they may equally well apply to a body of water with an envelope S of the same shape as the surface of the body. Hence the vertical motion of the center of gravity of a proportional body is the same as the vertical motion of the centroid of the displaced water in the absence of the body.

More generally, if we consider a very rigid structure of arbitrary density, then all modes of motion should be negligible except those which describe vertical translation and rotation, and these modes cannot be appreciably coupled to the flexure modes. Assume sufficient symmetry to eliminate coupling between translation and rotation, and denote the translation mode by the subscript 0. Then, effectively the body acts as a proportional solid for this mode, and

$$\psi_0 = 1, \quad \psi_0'' = \frac{(M_0 + L_0)^{\frac{1}{2}}}{M}$$

$$q' = q\,(M_0 + L_0)^{\frac{1}{2}}$$

Substituting in [45], we get

$$\ddot{q}_0 = \left[\zeta \int_0^l \frac{m\,dx}{M_0\, r^3} + \frac{\zeta}{M_0 + L_0} \int_0^l \frac{A\rho - m}{r^3}\,dx\right] \frac{\ddot{V}_g}{4\pi} \qquad [49]$$

Considering only the case for which $r \gg l$

$$\ddot{q}_0 = \left[\frac{\zeta}{r^3} + \frac{1}{M_0 + L_0}\,\frac{\zeta}{r^3}\,(M_w - M_0)\right] \frac{\ddot{V}_g}{4\pi} = \frac{\zeta}{4\pi r^3}\,\frac{M_w + L_0}{M_0 + L_0}\,\ddot{V}_g \qquad [50]$$

where M_w is the mass of the displaced water. Since everything starts from rest, the displacement is simply proportional to the bubble volume. This simple result could have been derived much more directly, without any reference to elastic motions, by basing it on the equations for the translation of a solid in a fluid.[2]

FLEXURAL MOTIONS OF A FLOATING BODY

The problem of the flexural reactions of a floating body to a pulsating bubble can be treated by the same methods described for the submerged body. An added complication is that, besides the other boundary conditions, the velocity potential Φ must vanish at the free surface. This condition is necessary in order to approximately satisfy the condition that the pressure at the free surface must always equal atmospheric pressure.

The condition that $\Phi_i = \Phi_g = 0$ at all points on the free surface may be satisfied by the usual device of constructing an image space which is the reflection of the fluid space in the plane of the free surface, and where the potential at any point is the negative of the potential at the corresponding point in the fluid. Hence if S' is the image of the underwater surface S and G' is the image of the bubble G (see Figure 2), then Φ_g and ϕ_i must satisfy the additional boundary conditions

$$-\frac{\partial \Phi_g}{\partial n} = v_g \quad \text{on } G' \qquad [51]$$

$$-\frac{\partial \Phi_g}{\partial n} = 0 \quad \text{on } S' \qquad [52]$$

$$-\frac{\partial \phi_i}{\partial n} = \psi_i \cos(n,k) \text{ on } S' \qquad [53]$$

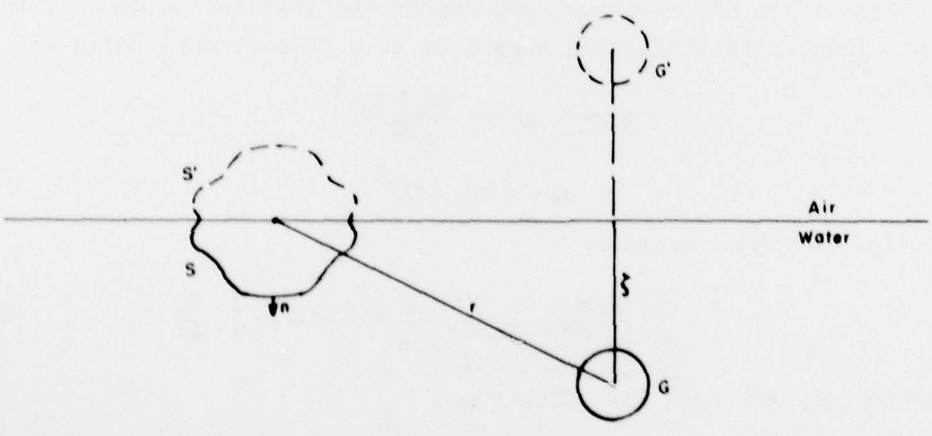

Figure 2 - Geometry for a Floating Body

where

$$v_g \text{ on } G = -v_g \qquad \text{for image point on } G' \qquad [54]$$

and

$$\cos(n,k) \text{ on } S = -\cos(n,k) \qquad \text{for image point on } S' \qquad [55]$$

Then the same analysis as before leads to the same differential equation, as in [37]:

$$\ddot{q}_i' + \omega_i'^2 q_i' = -\rho\phi_{ig}' \ddot{V}_g = -\rho \int \dot{\phi}_g \psi_i' \cos(n,k)\, d\sigma \qquad [56]$$

and

$$L_{ii}' = +\rho \int_S \phi_i' \psi_i' \cos(n,k)\, d\sigma$$

is defined as an integral over the underwater area only. Likewise the generalized force on the right side of the equation is computed by integrating over S, the underwater area, only.

However, in order to evaluate ϕ_{ig}' by Green's function

$$4\pi\phi_{ig}' = \int_{S+S'} \left[\phi_i' \frac{\partial}{\partial n}\left(\frac{1}{r_g}\right) - \frac{1}{r_g} \frac{\partial \phi_i'}{\partial n}\right] d\sigma \qquad [57]$$

the integration must be made over S and S'. This leads to a factor two in the equation for ϕ_{ig}' under conditions analogous to [44], i.e.,

$$4\pi\phi_{ig}' = \frac{2\zeta}{\rho} \int_0^l \frac{m\psi_i'' \, dx}{r^3} + \frac{2\zeta}{\rho} \int_0^l \frac{\psi_i'(A\rho - m)}{r^3}\, dx \qquad [58]$$

where A is now the area of the underwater section of the body at a distance x from one end.

As a special example, we give the equation for the vertical flexural vibrations of a floating proportional body of uniform density:

$$\ddot{q}_i + \omega_i^2 q_i = \left[\frac{2\rho\zeta}{4\pi M_i} \int_0^l \frac{A\psi_i \, dx}{r^3}\right] \ddot{V}_g \qquad [59]$$

Note that the only difference as compared with [48] is the factor of two in the generalized force.

MOTION OF THE BUBBLE

In order to solve the differential equations for the motions of the body, we must know the volume acceleration of the bubble $\ddot{V}_g$, or an equivalent. In a practical case this might be known from prior knowledge, e.g., from

photographs of the bubble motion. However, the equation for the motion of the bubble may also be obtained from this analysis.

We shall assume that the bubble is spherical at all times, of radius $R(t)$, and that near the bubble the velocity potential is given by

$$\Phi_g = \frac{R^2 \dot{R}}{r_g} = \frac{\dot{V}_g}{4\pi r_g}, \qquad [60]$$

and

$$\Phi_{gg} = R\dot{R} = \frac{\dot{V}_g}{4\pi R} \qquad [61]$$

Φ_g is a harmonic function which satisfies the boundary condition [11] at the bubble, but not the boundary condtion [12] at the body. It is clear that near the body the potential must be augmented by a term representing a "reflected" potential. We assume that the body is sufficiently distant from the bubble that this reflection term is negligible at the bubble itself. Then, substituting [61] into [25] and [26],

$$T = \sum_i \frac{M_i}{2} \dot{q}_i^2 + \sum_{i,j} \frac{L_{ij}}{2} \dot{q}_i \dot{q}_j + 2\pi\rho R^3 \dot{R}^2 + \sum_i 4\pi\rho \dot{q}_i \phi_{ig} R^2 \dot{R} \qquad [62]$$

$$V = \sum_i \frac{M_i}{2} \alpha_i^2 q_i^2 + \frac{4}{3}\pi R^3 p_h + E'(R) \qquad [63]$$

and applying Lagrange's equations to the coordinate R,

$$\sum_i 4\pi \ddot{q}_i \rho \phi_{ig} R^2 + 4\pi\rho R^3 \ddot{R} + 6\pi\rho R^2 \dot{R}^2 + 4\pi R^2 p_h + \frac{\partial E'}{\partial R} = 0 \qquad [64]$$

The first term couples the pulsation of the bubble to the motion of the solid. This term may be written as $4\pi R^2 p_{SG}$ where

$$p_{SG} = \sum_i \rho \dot{\phi}_{ig} = \sum_i \rho \ddot{q}_i \phi_{ig}$$

is the pressure at G due to the vibration of S. Again if the body is sufficiently distant, then p_{SG} is small compared for example with p_h which enters into the fourth term. If this first term is neglected, then the remainder of Equation [64] is equivalent to the differential equation which is usually derived for a bubble in an infinite fluid.[1] The equation must be solved either by numerical methods, or the main characteristics of the pulsation can be estimated by methods which are described in the literature.[3]

EFFECT OF AN INITIAL SHOCK WAVE

The foregoing was all based on the assumption that the flow which is generated by the bubble pulsation is an incompressive motion. This is a common assumption in bubble theory and is known to be accurate during a large part of the bubble oscillation period when the radial velocity of the bubble surface is small compared with c, the velocity of sound in water. However, in the initial stages of its motion, when, for example, a bubble is created by the detonation of a charge, a compressive wave is emitted and the velocity potential differs markedly from a harmonic function. We shall assume that this flow satisfies the wave equation

$$\nabla^2 \phi_g = \frac{1}{c^2} \frac{\partial^2 \phi_g}{\partial t^2} \qquad [65]$$

and we shall determine under what conditions the incompressive analysis remains valid. It is still assumed that the flows described by ϕ_i are incompressive.

Lagrange's equations can no longer be applied to the motion of the water, because it has been demonstrated only that they are applicable when the flow is incompressive.[3] We return, therefore, to the mode functions which describe the normal vibrations in air, and consider the forced vibration in the i^{th} mode due to an applied pressure of magnitude

$$\rho \dot{\phi} = \rho \dot{\phi}_g + \sum_j \rho \ddot{q}_j \phi_j \qquad [66]$$

The generalized force for this mode must be computed by integrating this pressure over S with $-\psi_i \cos(n, k)$ as a weighting function. Hence

$$\begin{aligned} M_i [\ddot{q}_i + \sigma_i^2 q_i] &= -\int_S \rho \dot{\phi}_g \psi_i \cos(n,k)\, d\sigma + \sum_j \rho \ddot{q}_j \int \phi_j \psi_i \cos(n,k)\, d\sigma \\ &= -\rho \frac{d}{dt} \int_S \phi_g \psi_i \cos(n,k)\, d\sigma - \sum_j L_{ij} \ddot{q}_j \end{aligned} \qquad [67]$$

Again we use Green's second identity to transform the first term on the right

$$-\int_S \phi_g \psi_i \cos(n,k)\, d\sigma = -\int_G \phi_i v_g\, d\sigma + \int \phi_i \nabla^2 \phi_g\, d\tau \qquad [68]$$

where the last integral is taken over the whole volume of water. Hence

$$\begin{aligned} -\rho \int \phi_g \psi_i \cos(n,k)\, d\sigma &= -\rho\, \phi_{ig} \frac{dV_g}{dt} + \rho \int \frac{\phi_i}{c^2} \ddot{\phi}_g\, d\tau \\ &= -\rho \phi_{ig} \frac{dV_g}{dt} + \int \frac{\phi_i}{c^2} \dot{p}\, d\tau \end{aligned} \qquad [69]$$

where $p(\xi,\eta,\zeta,t)$ is the local excess pressure due to the bubble. Substituting in [67]

$$M_i\ddot{q}_i + \sum_j L_{ij}\ddot{q}_j + M_i\alpha_i^2 q_i = -\rho\phi_{i0}\ddot{V}_0 + \int \frac{\phi_i}{c^2}\ddot{p}\,d\tau \qquad [70]$$

Now integrate twice with respect to time, starting with time zero when everything is at rest and the bubble is created:

$$M_i\left[q_i + \alpha_i^2\int_0^t\int_0^t q_i\,dt\,dt\right] + \sum_j L_{ij}q_j = -\rho\phi_{i0}V_0 + \int\frac{\phi_i}{c^2}p\,d\tau \qquad [71]$$

Consider the integrand of the last term. ϕ_i is a function which drops off as $\frac{\zeta}{r^3}$ from the target (see Equation [45]). Also it is of opposite sign above and below the target. $p(\xi,\eta,\zeta,t)$ is initially very large at the bubble, but this phase travels off into the water, the amplitude decaying as $\frac{1}{r}$. Furthermore it is known that p becomes negative at the bubble at about 11 percent of the bubble period and this negative phase travels out into the water. Hence it is clear that shortly after the creation of the bubble, a time t will exist at which the last term in Equation [71] will be negligible, or even zero, compared with the first term on the right, and after this time the last term will remain negligible. If also this time t is small compared with the natural period of oscillation in the i^{th} mode, i.e., if $\alpha^2t^2 \ll 4\pi^2$, then the integral term on the left side of this equation must be negligible compared with the first term. Hence at this time t, the displacement q_i, as calculated from Equation [71], will be the same whether or not the compressibility term is included. Furthermore, after this time the compressibility effect is known to be negligible and need not be included.

This result means that the incompressive theory is adequate to explain the flexural vibrations of the body even though the solid is initially exposed to a compressive shock wave, provided that the period of oscillation is long compared with the duration of the compressive wave. On the other hand, for high-frequency motions of the body the generalized forces are not related to $\ddot{V}_0$ in a simple way, and the theory is not applicable.

SUMMARY AND CONCLUSIONS

We may sum up the results of this analysis as follows.

1. When a flexible body is submerged in water, the modes of flexure in which the body vibrates become modified in two ways: (a) The water increases the inertia of the system and so decreases the natural frequency,

and (b) the inertia of the water couples together the various modes of flexure. However there does exist a new set of flexure modes in each of which the body may vibrate independently. The equations for these new modes are derived in terms of the old.

2. When this flexible body is exposed to a pulsating bubble in the liquid, the bubble motion is affected in two ways: (a) The presence of the body will modify the pressure field from the bubble and thus set up a pressure gradient at the bubble which will cause the bubble to migrate to the body, and (b) the motion of the body will generate a fluctuating pressure at the bubble which will modify the pulsation. The first effect has been ignored; an equation has been derived for the second effect.

3. The bubble will generate flexural motions in the body. An equation is derived which relates the motion in each normal mode of flexure to the volume acceleration of the bubble. This equation is applicable whenever the bubble is sufficiently distant from the body so that (a) the normal modes of the body are not changed by the presence of the bubble, (b) the migration is negligible, and (c) the distance between bubble and body is large compared with the section diameter of the body. In the event the body has some special symmetry or "proportional" structure, a simplified form of these equations is derived.

4. The theory is based upon the assumption that the flow associated with the bubble motion is incompressive. Yet the initial stage of the motion may involve a highly compressive wave motion. Nevertheless it is shown that the analysis remains applicable provided the duration of the compressive wave is small compared with (a) the period of pulsation of the bubble and (b) the period of vibration of the body. Another effect of the compressive wave, which has not been discussed, is that it provides a mechanism by which bubble energy is radiated away from the system.

It is also assumed implicitly that the compressibility of the water has negligible effect upon the flexural vibration modes of the body. However, it is clear that if the normal velocities of the surface are not small compared with the velocity of sound in water, then we may expect that the effect of the water on the normal modes is not only to increase the inertia of the system but also to provide a mechanism for energy dissipation. The sound waves radiated by the vibration will drain energy from the body and thus dampen the vibration.

The experimental verification of the analysis will be discussed in a subsequent report, as well as some quantitative implications of the variation of flexure with target and charge characteristics.

APPENDIX

THE RELATIONS BETWEEN THE FREE FLEXURAL MODES IN WATER AND IN AIR

We derive here some explicit relations between the normal modes and frequencies of a body in water and the corresponding modes and frequencies in air. These relations are most conveniently obtained in matrix notation. We will write a matrix in bold face $\mathbf{A}$, its transpose will be written $\tilde{\mathbf{A}}$ and its inverse as $\mathbf{A}^{-1}$.

Let $\mathbf{q}$ denote the column matrix with elements $q_i(t)$,
$\boldsymbol{\psi}$ denote the column matrix with elements $\psi_i(x)$,
$\boldsymbol{\phi}$ denote the column matrix with elements ϕ_i,
and be defined only on surface of the body,
$\mathbf{M}$ denote the diagonal matrix with elements M_i,
$\mathbf{L}$ denote the symmetric matrix with elements L_{ij}, and
$\boldsymbol{\alpha}$ denote the diagonal matrix with elements α_i.

Then from Equations [25] and [26] we can write for the kinetic energy and the potential energy of the system in the absence of the bubble

$$2T = \dot{\tilde{\mathbf{q}}} \left(\mathbf{M} + \mathbf{L}\right) \dot{\mathbf{q}} = \dot{\tilde{\mathbf{q}}}\, \mathbf{M}^{\frac{1}{2}} \boldsymbol{\alpha} \left[\mathbf{M}^{-\frac{1}{2}} \boldsymbol{\alpha}^{-1} \left(\mathbf{M} + \mathbf{L}\right) \boldsymbol{\alpha}^{-1} \mathbf{M}^{-\frac{1}{2}}\right] \boldsymbol{\alpha} \mathbf{M}^{\frac{1}{2}} \dot{\mathbf{q}} \quad [72]$$

$$2V = \tilde{\mathbf{q}}\, \mathbf{M} \boldsymbol{\alpha}^2 \mathbf{q} = \left(\tilde{\mathbf{q}}\, \mathbf{M}^{\frac{1}{2}} \boldsymbol{\alpha}\right) \left(\boldsymbol{\alpha} \mathbf{M}^{\frac{1}{2}} \mathbf{q}\right) \quad [73]$$

where the insertions have been made in order to express $\mathbf{V}$ as a sum of squares. Now diagonalize the matrix in brackets by means of an orthogonality transformation:

$$\left[\mathbf{M}^{-\frac{1}{2}} \boldsymbol{\alpha}^{-1} \left(\mathbf{M} + \mathbf{L}\right) \boldsymbol{\alpha}^{-1} \mathbf{M}^{-\frac{1}{2}}\right] = \mathbf{R}^{-1} \boldsymbol{\Omega}^{-2} \mathbf{R} \quad [74]$$

where $\boldsymbol{\Omega}$ is a matrix such that $\boldsymbol{\Omega}^{-2}$ is a diagonal matrix whose elements (ω_i^{-2}) are the characteristic roots of the matrix in brackets, and $\mathbf{R}$ is an orthogonal matrix which is compounded of the characteristic solutions of the matrix in brackets. Hence

$$2T = \left(\dot{\tilde{\mathbf{q}}}\, \mathbf{M}^{\frac{1}{2}} \boldsymbol{\alpha} \mathbf{R}^{-1}\right) \boldsymbol{\Omega}^{-2} \left(\mathbf{R} \boldsymbol{\alpha} \mathbf{M}^{\frac{1}{2}} \dot{\mathbf{q}}\right) = \dot{\tilde{\mathbf{q}}}' \dot{\mathbf{q}}' \quad [75]$$

$$2V = \left(\tilde{\mathbf{q}}\, \mathbf{M}^{\frac{1}{2}} \boldsymbol{\alpha} \mathbf{R}^{-1} \boldsymbol{\Omega}^{-1}\right) \boldsymbol{\Omega}^2 \left(\boldsymbol{\Omega}^{-1} \mathbf{R} \boldsymbol{\alpha} \mathbf{M}^{\frac{1}{2}} \mathbf{q}\right) = \tilde{\mathbf{q}}' \boldsymbol{\Omega}^2 \mathbf{q}' \quad [76]$$

where we now have a new set of normal coordinates

$$\mathbf{q}' = \left(\boldsymbol{\Omega}^{-1} \mathbf{R} \boldsymbol{\alpha} \mathbf{M}^{\frac{1}{2}}\right) \mathbf{q} \equiv \mathbf{A} \mathbf{q} \quad [77]$$

where $\mathbf{A}$ is the transformation matrix in parenthesis. Hence V and T are both now sums of squares, the modes are independent, and the new natural frequencies are the inverse square roots of the characteristic roots of the matrix in [74]. Also we define an associated set of mode functions and potentials

$$\boldsymbol{\psi}' = \tilde{\mathbf{A}}^{-1}\boldsymbol{\psi} \; ; \; \boldsymbol{\phi}' = \tilde{\mathbf{A}}^{-1}\boldsymbol{\phi} \tag{78}$$

and substituting in [72]

$$2T = \dot{\tilde{\mathbf{q}}}\left(\mathbf{M} + \mathbf{L}\right)\dot{\mathbf{q}} = \dot{\tilde{\mathbf{q}}}'\mathbf{M}'\dot{\mathbf{q}}' + \dot{\tilde{\mathbf{q}}}'\mathbf{L}'\dot{\mathbf{q}}' = \dot{\tilde{\mathbf{q}}}'\dot{\mathbf{q}}' \tag{79}$$

where

$$\mathbf{M}' = \tilde{\mathbf{A}}^{-1}\mathbf{M}\mathbf{A}^{-1} = \int_0^l \boldsymbol{\psi}'\tilde{\boldsymbol{\psi}}'\, m\, dx \tag{80}$$

and

$$\mathbf{L}' = \tilde{\mathbf{A}}^{-1}\mathbf{L}\mathbf{A}^{-1} = +\rho\int_S \boldsymbol{\phi}'\tilde{\boldsymbol{\psi}}' \cos(n, k)\, d\sigma \tag{81}$$

and

$$\mathbf{M}' + \mathbf{L}' = \mathbf{I} \quad \text{the unit matrix.} \tag{82}$$

This completes the specification of the new coordinates and parameters as unique functions of the old coordinates and parameters.

There exists a relation between the new potentials and the new mode functions which is useful in the analysis. We define a new series of functions $\bar{\phi}_i'(x)$ as an integral of ϕ_i' around the perimeter s of a section of the body at the distance x from one end, i.e.,

$$\bar{\phi}_i'(x) = \oint \phi_i' \cos(n, k)\, ds \tag{83}$$

Let $\bar{\boldsymbol{\phi}}'$ denote the column matrix with elements $\bar{\phi}_i'(x)$ and consider the relation between $\bar{\boldsymbol{\phi}}'$ and $\boldsymbol{\psi}'$. Now since the functions ψ_i, and also ψ_i', are assumed to form a complete set, the function $\dfrac{\bar{\phi}_i'(x)}{m(x)}$ can be expanded in terms of $\psi_j(x)$ and we can take

$$\bar{\boldsymbol{\phi}}' = m(x)\,\mathbf{B}\,\boldsymbol{\psi}' \tag{84}$$

where $\mathbf{B}$ is an undetermined matrix with constant coefficients. Now postmultiply both of above by the matrix $\tilde{\boldsymbol{\psi}}'$ and integrate with respect to x.

$$\int_0^l \bar{\boldsymbol{\phi}}'\tilde{\boldsymbol{\psi}}'\, dx = \mathbf{B}\int_0^l \boldsymbol{\psi}\boldsymbol{\psi}'\, m\, dx$$

$$\frac{L}{\rho} = BM' \qquad [85]$$

and from [81] and [80] and substituting in [84]

$$\bar{\Phi}' = \frac{m}{\rho} L' M'^{-1} \phi' = \frac{m}{\rho} \left(I - M' \right) M'^{-1} \phi' \qquad [86]$$

$$\bar{\Phi}' = \frac{m}{\rho} \left(\psi'' - \psi' \right) \qquad [87]$$

where

$$\psi'' = M'^{-1} \psi' \qquad [88]$$

and represents a column matrix of mode functions which are particular combinations of the mode functions ψ_i'. This final result is an expression for the potential distribution on the surface of the body as a function of the motions which generate the potential. It is interesting that, in general, the potential $\overline{\phi_i}'$ does not depend on the single mode ψ_i', but also on all the modes ψ_j', $j \neq i$. This dependence on all the modes enters through the presence of ψ_i'' in [87].

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